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SOLUTICN OF PRCBLEMS IN NUMBER THREE.

SOLUTIONS of problems in No. 3 have been received as follows:

From Marcus Baker, 203, 209; Geo. M. Day, 204, 209; Prof. W. W. Hendrickson, 204; H. Heaton, 202, 207, 208; W. E. Heal, 208, 209, 210; Prof. J. H. Kershner, 202, 203, 204, 205, 208, 209; Prof. H. T. J. Ludwick, 204; Prof. D. J. Mc Adam, 205, 209; W. V. Mc Knight, 202, 203, 205, 209; Artemas Martin, 203, 204, 205, 208, 209, 210; P. Richardson, 209; E. B. Seitz, 202, 203, 204, 205, 207, 208, 209; Prof. J. Scheffer, 202, 203, 205, 206, 208, 209.

By an oversight, in No. 3, we failed to credit Prof. Kershner for a solution of problem 200. His solution was substantially the same as the published solution.

The query by Prof. Mc Adam was answered, in nearly the same way, by the querist, and by Prof. Scheffer, Mr. Mc Knight, and Mr. Seitz.

No answer to the second query on p. 96 has been received.

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202. "If $r \div d$ reduces to a repetend, prove that $s \div d = R \div 9$, where s is the sum of the remainders occurring in the division, and R is the sum of the digits of the repetend."

SOLUTION BY PROF. J. SCHEFFER.

Denoting the successive figures of the circulator by $x, y, z, \ldots u$, and the successive remainders by $m_1, m_2, m_3, \ldots m_n$, we obviously have the relation:

$$\frac{r}{d} = \frac{x + (m_1 \div d)}{10},$$

$$\frac{m_1}{d} = \frac{y + (m_2 \div d)}{10},$$

$$\frac{m_2}{d} = \frac{z + (m_3 \div d)}{10},$$

$$\vdots \qquad\qquad \vdots$$

$$\frac{m_n}{d} = \frac{u + (r \div d)}{10}.$$

Adding we have $$\frac{s}{d} = \frac{R + (s \div d)}{10}; \text{ whence } \frac{9s}{d} = R, \text{ or } \frac{s}{d} = \frac{R}{9}.$$

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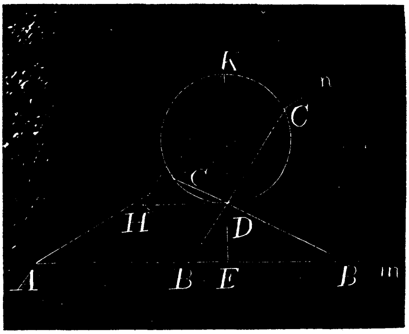
203. "Two lines Am and An make a given angle at A, and D is a point given in position between Am and An. Draw a straight line BC through D so that $BD \times DC =$ a given quantity, B and C being points in the lines Am and An, respectively."

SOLUTION BY E. B. SEITZ, GREENVILLE, OHIO.



Draw DE perpendicular to Am; from D draw DH parallel to Am, and let the square of the length of DH be equal to the given quantity. Draw HK perpendicular to HE, meeting ED produced in K; on DK as diameter describe a circle, cutting An in C and C'. Through D draw BC and $B'C'$, either of which is the line required.

For, in the similar right angled triangles BDE, CDK and EDH, HDK, we have $BD\times DC = ED\times DK = HD^2 =$ the given quantity. The proof for $B'C'$ is similar.

204. "Find the locus of the intersection of normals to the ellipse which are perpendicular to each other."

SOLUTION BY PROF. W. W. HENDRICKSON, ANNAPOLIS, MD.

The equation to the normal is

$$y = mx - \frac{m(a^2-b^2)}{\sqrt{(b^2m^2+a^2)}}, \qquad \text{whence}$$

$$\left.\begin{aligned} &m^4 - \frac{2y}{x}m^3 + \frac{b^2y^2+a^2x^2-(a^2-b^2)^2}{b^2x^2}m^2 - \frac{2a^2y}{b^2x}m + \frac{a^2y^2}{b^2x^2} = 0, \\ \text{or}\quad &m^4 + Pm^3 + Qm^2 + Rm + S = 0. \end{aligned}\right\} \qquad (1)$$

Since the product of two roots of this equation is -1, one of its quadratic factors may be assumed as m^2+Am-1, and the other as m^2+Bm+C; then the equation

$$m^4+(A+B)m^3+(AB+C-1)m^2+(AC-B)m-C=0$$

is identical with (1), hence

$$C = -S,\ AC-B = R,\ A+B = P \qquad (2)$$

and
$$AB+C-1 = Q. \qquad (3)$$

Substituting in (3) the values of A, B and C, obtained from (2), we find

$$(Q+S+1)(S-1)^2+(P+R)(R+PS) = 0;$$

and, replacing P, Q, R and S by their values, we have, after reduction,

$$(a^2+b^2)(x^2+y^2)(a^2y^2+b^2x^2)^2 = (a^2-b^2)^2(a^2y^2-b^2x^2)^2.$$

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205. "The distance between the centres of two circles is given, and the radius of one, fiind the radius of the other, when the length of the endless band which passes around them is given."

SOLUTION BY E. B. SEITZ.

Let a equal the distance between the centers of the circles, b equal the length of the endless band, r, the radius of the given circle, x, the unknown radius, and θ, the angle formed by the line joining the centers and the radius of the given circle drawn to the point of contact of the common external tangent. Then we have

$$2a\sin\theta + 2r(\pi - \theta) + 2(r - a\cos\theta)\,\theta = b, \qquad (1)$$

$$x = r - a\cos\theta. \qquad (2)$$

From (1)

$$\sin\theta - \theta\cos\theta = (b - 2\pi r) \div 2a. \qquad (3)$$

From (3) by means of a table of natural sines and cosines we can find, by the method of approximation, the value of θ, and then from (2) we can find x.

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206. "Determine the position of a plane from the measured rectangular coordinates of n points, each measure being equally good."

SOLUTION BY PROF. J. SCHEFFER, COLLEGE OF ST. JAMES, MD.

We denote the co-ordinates of the n given points by x', y', z'; x'', y'', z''; x''', y''', z'''; etc.: and the equation of the plane by

$$Ax + By + Cz + D = 0,$$

in which the coefficients A, B, C and D are to be determined by means of the method of least squares. By substituting the above values we have

$$\begin{aligned} Ax' + By' + Cz' + D &= 0, \\ Ax'' + By'' + Cz'' + D &= 0, \\ Ax''' + By''' + Cz''' + D &= 0, \\ \dots\dots\dots\dots\dots\dots \end{aligned}$$

Now we multiply the first of these equations by x', the second by x'', the third by x''', etc.; and add up. Then we again multiply the first by y', the second by y'', the third by y''', etc., and add up. Again multiply the first by z', the second by z'', the third by z''', etc., and add up; and again multiply the first, second, third, etc. by 1, and add up. Whence we get, denoting for the sake of brevity,

$x'^2 + x''^2 + x'''^2 + \dots$ by $S(x^2)$,
$x'y' + x''y'' + x'''y''' + \dots$ by $S(xy)$,
$x'z' + x''z'' + x'''z''' + \dots$ by $S(xz)$,
$y'z' + y''z'' + y'''z''' + \dots$ by $S(yz)$,
$x' + x'' + x''' + \dots$ by $S(x)$,
$y' + y'' + y''' + \dots$ by $S(y)$,
$z' + z'' + z''' + \dots$ by $S(z)$, the final equations,

$$A.S(x^2)+B.S(xy)+C.S(xz)+D.S(x)=0,$$
$$A.S(xy)+B.S(y^2)+C.S(yz)+D.S(y)=0,$$
$$A.S(xz)+B.S(yz)+C.S(z^2)+D.S(z)=0,$$
$$A.S(x\ )+B.S(y\ )+C.S(z\ )+D.n\quad=0;$$

whence, by elimination, we can derive the values of A, B, C and D.

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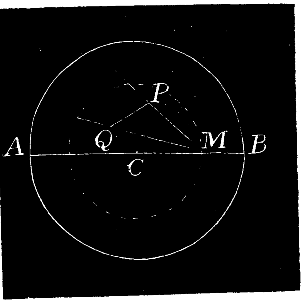
207. "If M, N, P, Q are four random points in the surface of a circle, find the chance that E, the intersection of the straight lines through M, N and P, Q, lies between M, N and between P, Q."

SOLUTION BY HENRY HEATON, SABULA, IOWA.



Put p = the required probability, and q = the chance that the point N will fall within the triangle MPQ. Then $4q$= the chance that one of the four points will fall within the triangle formed by the other three.

If no one of the points fall within the triangle formed by the other three, either MN will intersect PQ, MP will intersect NQ or MQ will intersect PN, either of which events is equally probable. Hence $p=\frac{1}{3}(1-4q)$.

In the figure put $CM=x$, $MQ=y$, $MP=z$, $\angle CMQ=\theta$, and $\angle CMP=\varphi$. Then the area of the triangle MPQ = $\frac{1}{2}yz\sin(\varphi-\theta)$, and

$$q=\frac{\int_0^1\int_{-\frac{1}{2}\pi}^{\frac{1}{2}\pi}\int_\theta^{\frac{1}{2}\pi}\int_0^{2x\cos\theta}\int_0^{2x\cos\phi}\frac{1}{2}zy\sin(\varphi-\theta)xdxd\theta d\varphi ydyzdz}{\int_0^1\int_{-\frac{1}{2}\pi}^{\frac{1}{2}\pi}\int_\theta^{\frac{1}{2}\pi}\int_0^{2x\cos\theta}\int_0^{2x\cos\phi}\pi xdxd\theta d\varphi ydyzdz}$$

$$=\frac{6}{\pi^2}\int_0^1\int_{-\frac{1}{2}\pi}^{\frac{1}{2}\pi}\int_\theta^{\frac{1}{2}\pi}\int_0^{2x\cos\theta}\int_0^{2x\cos\phi}zy\sin(\varphi-\theta)xdxd\theta d\varphi ydyzdz$$

$$=\frac{128}{3\pi^3}\int_0^1\int_{-\frac{1}{2}\pi}^{\frac{1}{2}\pi}\int_\theta^{\frac{1}{2}\pi}x^7\cos^3\theta\cos^3\varphi\sin(\varphi-\theta)dxd\theta d\varphi$$

$$=\frac{8}{3\pi^3}\int_0^1\int_{-\frac{1}{2}\pi}^{\frac{1}{2}\pi}x^7(6\cos^4\theta-2\cos^6\theta+6\theta\cos^3\theta\sin\theta-3\pi\cos^3\theta\sin\theta)dxd\theta$$

$$=\frac{35}{6\pi^2}\int_0^1x^7dx=\frac{35}{48\pi^2}.\quad\text{Hence }p=\frac{1}{3}-\frac{35}{36\pi^2}.$$

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208. "When the tangent, QT, to a fixed parabola is of constant length, and one extremity, T, moves along the axis to the origin, what is the curve described by the other extremity, Q?"

SOLUTION BY PROF. J. H. KERSHNER, MERCERSBERG, PA.

Let the rectangular coordinates of Q be (x_1, y_1), of P, (x_2, y_2), and let l be the length of QT, the given tangent.

By the similar triangles TPR_2, TQR_1, $TR_2 : PR_2 :: TR_1 : QR_1$, or

$$2x_2 : y_2 :: l^2—y_1^2 : y_1^2.$$

Putting $z^2 = l^2—y_1^2$, squaring, and substituting $2px_2$ for y_2, we get

$$2x_2 : p :: z^2 : y_1^2, \text{ or } 2z—x_1 : p :: z^2 : y_1^2.$$

$$\therefore\ xy^2+pz^2 = 2zy^2,$$

is the equation required, and the locus is one represented by an equation of the 6th degree.

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209. "C is the center of a circle of known radius, DB is a diameter produced and AB is an inflexible rod, of known length. One end, A, moves continually in the circumference of the circle while the other end, B, slides to and fro upon DB. Required the equation of the curve described by any point P, in the rod."

SOLUTION BY E. B.S EITZ.

Draw AN and PM perpendicular to DB, and put $AC=r$, $AB=a$, $BP=b$, $CM=x$, $PM=y$, and $\angle ABC = \theta$. Then

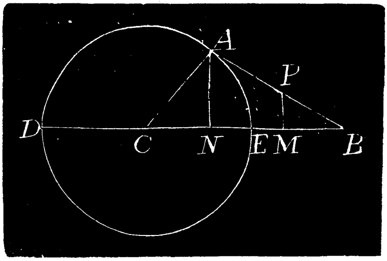
$$x=(a—b)\cos\theta \pm \surd(r^2—a^2\sin^2\theta), \quad (1)$$

$$y=b\sin\theta. \quad (2)$$

Substituting the values of $\sin\theta$ and $\cos\theta$ from (2) in (1) we have

$$bx=(a—b)\surd(b^2—y^2)\pm\surd(b^2r^2—a^2y^2),$$

the required equation, the double sign being taken + when A is in the fiast or fourth quadrant from E, and — when A is in the second or third quadrant. The curve is closd; its area is $\pi br^2\div a$.



[Mr. Baker, who obtains a similar equation to the above, remarks: "It is evident from this equation without further change that the curve is *not* a conic section. * * * It is readily traced by a simple mechanical device, and the curve so traced resembles a longitudinal section of a hen's egg."]

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210. "Find such integral values of x, y and z as will satisfy the conditions

$$x^2+axy+y^2=\square,$$
$$x^2+bxz+z^2=\square,$$
$$y^2+cyz+z^2=\square."$$

SOLUTION BY ARTEMAS MARTIN, M. A., ERIE, PA.

Assume $x = an^2 - 2mn$, $y = m^2 - n^2$, $z = cn^2 + 2mn$; then

$$x^2 + axy + y^2 = (m^2 - amn + n^2)^2,$$

$$y^2 + cyz + z^2 = (m^2 + cmn + n^2)^2.$$

But the second condition is not satisfied by the assumed values of x, y and z.

Let us put

$$an - 2m = r, \qquad (1)$$

$$cn + 2m = s; \qquad (2)$$

then $x = nr$, $z = ns$, $x^2 + bxz + z^2 = n^2(r^2 + brs + s^2)$.

Assume now

$$r = t(p^2 - q^2), \qquad (3)$$

$$s = t(bq^2 + 2pq), \qquad (4)$$

and we have $r^2 + brs + s^2 = t^2(p^2 + bpq + q^2)^2$, and consequently

$$x^2 + bxz + z^2 = n^2t^2(p^2 + bpq + q^2)^2.$$

From (1), (2), (3) and (4) we have

$$an - 2m = t(p^2 - q^2), \qquad (5)$$

$$cn + 2m = t(bq^2 + 2pq); \qquad (6)$$

which give

$$m = \frac{at(bq^2 + 2pq) - ct(p^2 - q^2)}{2(a+c)} = a(bq^2 + 2pq) - c(p^2 - q^2),$$

$$n = \frac{2t(bq^2 + 2pq) + 2t(p^2 - q^2)}{2(a+c)} = 2(bq^2 + 2pq) + 2(p^2 - q^2),$$

if we take $t = 2(a+c)$; where p and q may be any numbers that will give x, y, z all positive or all negative. See *Laybourn's Mathematical Repository*, New Series, Vol. 3, Part I, pp. 151—164, for elaborate general solutions to this problem.

QUERY. "In works on Practical Geometry, the following method of inscribing a regular polygon of n sides is given:

Let AB be a diameter of a circle. With A and B as centers and radius AB describe arcs intersecting in D. Divide AB into n equal parts. Draw DE through the second point of division; the arc AE is one-nth part of the circumference. Is there a general demonstration published?"

ANSWER BY E. B. SEITZ.

The method is strictly correct only for regular polygons of 3, 4 and 6 sides, which is shown as follows:

Let C be the center of the circle, F the second point of division; draw EM perpendicular to DC produced. Let $AC = r$, $CM = x$, and $\angle ACE = \varphi$.